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The number of even selections (2 at a time, 4 at a time, &c.)

$$=S'=\frac{n(n-1)}{2\,!}+\frac{n(n-1)(n-2)(n-3)}{4\,!}+\&c.\quad \text{Now } (1-1)^n=0$$

$$=1-n+\frac{n(n-1)}{2\,!}-\frac{n(n-1)(n-2)}{3\,!}+\frac{n(n-1)(n-2)(n-3)}{4\,!}-\&c.$$

The sum of the negative terms in this series $= S$, and the sum of the positive terms $= S'+1$. Hence $S = S' + 1$, or $S - S' = 1$. That is, the possible odd selections exceed the possible even selections, and therefore the chances are in favor of drawing an odd number."

SOLUTION OF PROBLEMS IN NUMBER FIVE.

SOLUTIONS of problems in number five have been received as follows:

From R. J. Adcock, 136; Marcus Baker, 125, 126, 127, 130 and 133; Henry Gunder, 125, 126, 127, 128, 130, 133 and 136; William Hoover, 125, 126, 130, 133 and 136; Henry Heaton, 125, 128 and 135; Christine Ladd, 129; Prof. H. T. J. Ludwick, 130, 135 and 136; Artemas Martin, 125, 133, 134 and 136; D. J. Mc. Adam, 125, 130 and 133; E. B. Seitz, 125, 127, 129, 130, 133, 134, 135 and 136; Prof. J. Scheffer, 125, 126, 127, 128, 129, 130 and 133.

125.—"From a point E, in a square field $ABCD$, lines drawn to the corners A, B and C are found to be 50, 30 and 40 rods, respectively. Required the side of the field."

SOLUTION BY E. B. SEITZ, GREENVILLE, OHIO.

Construct the right angled isosceles triangle BEF, making $BF = BE = b = 30$ rods; on EF construct the triangle AEF, making $AE = a = 50$ rods, and $AF = c = 40$ rods; and on AB construct the square $ABCD$, which is the square required; for we can easily prove that $EC = AF$.

Put $\angle AEF = \alpha$. Then we have $\cos\alpha = (a^2 + 2b^2 - c^2) \div (2ab\sqrt{2})$, and

$$\begin{aligned} AB &= \sqrt{[a^2 + b^2 - 2ab\cos(\alpha + \tfrac{1}{4}\pi)]} \\ &= \tfrac{1}{2}\sqrt{[2a^2 + 2c^2 + 2\sqrt{(4a^2b^2 + 4b^2c^2 + 2a^2c^2 - 4b^4 - a^4 - c^4)}]} \\ &= 5\sqrt{(82 + 6\sqrt{119})} = 60.71496 \text{ rods.} \end{aligned}$$

126.—"In a plane triangle are given the vertical angle A and the bisectors β and γ of the base angles B and C; determine the triangle."

SOLUTION BY PROF. J. SCHEFFER.

We easily obtain the proportions, $c : \beta = \sin(A+\frac{1}{2}B) : \sin A$; $b : \gamma = \sin(A+\frac{1}{2}C) : \sin A$. But since $c : b = \sin C : \sin B$ we get

$$\frac{\beta\sin(A+\frac{1}{2}B)}{\gamma\sin(A+\frac{1}{2}C)} = \frac{\sin C}{\sin B} = \frac{\sin(A+B)}{\sin B} = \frac{\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A+B)}{\sin\frac{1}{2}B\cos\frac{1}{2}B}$$

But $\sin(A+\frac{1}{2}C) = \cos\frac{1}{2}(A-B)$, $\therefore\ \beta\sin(A+\frac{1}{2}B)\sin\frac{1}{2}B\cos\frac{1}{2}B = \gamma\cos\frac{1}{2}(A-B)\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A+B)$, or $\beta\sin\frac{1}{2}B\cos\frac{1}{2}B(\sin A\cos\frac{1}{2}B + \cos A\sin\frac{1}{2}B) = \gamma(\sin\frac{1}{2}A\cos\frac{1}{2}B + \cos\frac{1}{2}A\sin\frac{1}{2}B)(\cos^2\frac{1}{2}A\cos^2\frac{1}{2}B - \sin^2\frac{1}{2}A \times \sin^2\frac{1}{2}B)$, or, dividing by $\cos\frac{1}{2}B.\cos^2\frac{1}{2}B$, and putting, for brevity, $\sin\frac{1}{2}A = m$, $\cos\frac{1}{2}A = n$ and $\tan\frac{1}{2}B = x$, we obtain, after reduction, the cubic eq'n,

$$x^3 + \frac{(n^2-m^2)\beta + m^3\gamma}{m^2n\gamma}x^2 + \frac{2m\beta - n^2\gamma}{m^2\gamma}x + \frac{n}{m} = 0.$$

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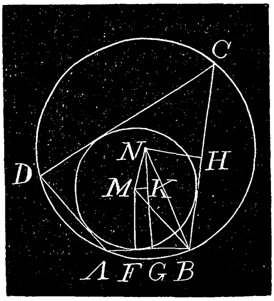
127. "Denoting the radii of the inscribed and circumscribed circles in and about a quadrilateral by r and R, and the distance between the two centres by h, prove the relation:

$$\left(\frac{r}{R+h}\right)^2 + \left(\frac{r}{R-h}\right)^2 = 1."$$

SOLUTION BY E. B. SEITZ.

Let M, N be the centres of the inscribed and circumscribed circles. Put $AB = a$, $BC = b$, $CD = c$, $DA = d$, $MF = r$, $NB = R$, $MN = h$, $\angle ABC = B$.



Then we have area $ABCD = \sqrt{(abcd)}$, $BG = \frac{1}{2}a$, $BH = \frac{1}{2}b$, $BF = R\cot\frac{1}{2}B$, $NG = \frac{1}{2}b\operatorname{cosec}B - \frac{1}{2}a\cot B$, $r = \frac{\sqrt{(abcd)}}{a+c}$, $\sin B = \frac{2\sqrt{(abcd)}}{ab+cd}$, $\cos B = \frac{ab-cd}{ab+cd}$, $\cot\frac{1}{2}B = \sqrt{\left(\frac{ab}{cd}\right)}$, $a+c = b+d$, . . (1)

$\frac{1}{4}(b\operatorname{cosec}B - a\cot B)^2 + \frac{1}{4}a^2 = R^2$, ... (2)

and $(\frac{1}{2}b\operatorname{cosec}B - \frac{1}{2}a\cot B - r)^2 + (r\cot\frac{1}{2}B - \frac{1}{2}a)^2 = h^2$. . . . . . . (3)

Subtracting (2) from (3), substituting the value of $\cot\frac{1}{2}B$, and reducing, we find $R^2 + r^2 - h^2 = r[b\operatorname{cosec}B - a\cot B + (ar \div c)]$. . . . . . . (4)

Squaring (4), adding (2) $\times\ 4r^2$, transposing and reducing, we have

$$(R^2+r^2-h^2)^2 = r^2\left[4R^2+r^2-a^2+\frac{(a^2-c^2)r^2}{c^2}+\frac{2abr}{c\sin B}-\frac{2a^2r\cos B}{c\sin B}\right] \quad ..(5)$$

By substituting for r, $\sin B$ and $\cos B$, in the last three terms in (5), we have

$$\frac{abd(a-c)}{c(a+c)} + \frac{ab(ab+cd)}{c(a+c)} - \frac{a^2(ab-cd)}{c(a+c)} = \frac{a^2}{c(a+c)}\Big[b(b+d) - ab + cd\Big] = a^2.$$

Hence (5) reduces to $(R^2+r^2-h^2)^2 = 4R^2r^2 + r^4$. . . . . (6)

By reduction (6) becomes $(R^2—h^2)^2 = 2r^2(R^2+h^2) = r^2(R+h)^2+r^2(R—h)^2$. Dividing this equation by $(R^2—h^2)^2$, we get

$$\left(\frac{r}{R+h}\right)^2 + \left(\frac{r}{R-h}\right)^2 = 1.$$

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128. "Prove the identity of

$$\frac{n(n-1)(n-2)(n-3)\dots1}{(m+1)(m+2)\dots(m+n+1)} \text{ and } \frac{1}{m+1} - \frac{n}{m+2} + \frac{n(n-1)}{1.2(m+3)} - \dots$$
$$+(-1)^n \frac{1}{(m+n+1)},$$

n being a positive integer."

SOLUTION BY HENRY GUNDER NORTH MANCHESTER, IND.

If the integral, $\int_0^1 x^m(1-x)^n dx$, be taken *by parts*, and again by expanding $(1-x)^n$ in a series and integrating each term, the results are respectively the two members of the given equation; hence their equality.—See Todhunter's Integral Calculus, pp. 70 and 71.

[We have an elegant solution of 129, by E. B. Seitz, but, for want of room, are compelled to defer its publication to No. 1, Vol. IV.]

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130. "Required the dimensions of an open cylindrical vessel of a given capacity so that the smallest possible quantity of metal shall be used in its construction, its thickness being already determined upon."

SOLUTION BY MARCUS BAKER, U. S. COAST SURVEY.

Put r, h, c and t respectively equal the internal radius, internal height, capacity, and thickness of metal. Then is $c = \pi r^2 h$, whence $h = c \div \pi r^2$.

The amount of metal used is $\pi(r+t)^2(h+t) - c =$ a minimum, or
$(r+t)^2[(c\div\pi r^2) + t^2] = [1+(t\div r)]^2[(c\div\pi) + tr^2] =$ a minimum.

Differentiating, $2t[1+(t\div r)]^2 r - 2[(c\div\pi)+tr^2][1+(t\div r)]t\div r^2 = 0$;
$\therefore r^3[1+(t\div r)]-(c\div\pi)-tr^2=0$; $\therefore r = \sqrt[3]{(c\div\pi)} = c\div[\pi\sqrt[3]{(c\div\pi)^2}] = h$.

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131. "If a body be impelled by the force of a fluid having the velocity v_1, and if the force of the impelling fluid to move the body be as $mv + nv^2$, where v is the velocity of the fluid relative to the body, what must be the velocity of the body in order that the work performed on the body by the fluid, in a unit of time, may be a maximum?"

SOLUTION BY HENRY HEATON.

Ler x = the velocity of the body. Then the work performed is a max. when $x(mv+nv^2)$ is a maximum. $v = v_1 - x$;

$$\therefore\ x(mv+nv^2) = nx^3 - (2nv_1+m)x^2+(mv_1+nv_1^2)x = u.$$

$$\frac{du}{dx} = 3nx^2 - 2(2nv_1+m)x+mv_1+nv_1^2 = 0. \ldots\ldots\ldots (1)$$

$$\frac{d^2u}{dx^2} = 6nx - 2(2nv_1+m). \ldots\ldots\ldots (2)$$

From (1), $x = [2nv_1+m\pm\sqrt{(n^2v_1^2+mnv_1+m^2)}]\div 3n$. Hence for a max. or a minimum, x = respectively, $[2nv_1+m \mp \sqrt{(n^2v_1^2+mnv_1+m^2)}]\div 3n$.

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132. "Integrate the expression $\dfrac{x.dx}{(x^3+8)\sqrt{(x^3-1)}}$."

[No solution of this problem has been received. The expression $\dfrac{xdx}{(x^3+8)\sqrt{(x^3-1)}}$, may be written $\dfrac{xdx}{\sqrt{[(x^3+8)^2(x^3-1)]}} = \dfrac{xdx}{\sqrt{\varphi x}}$, where φx is a rational function of x. Hence, because the dimension of φx is more than 2, and may be represented by $2n+1$, the given expression is an *ultra elliptic function*, of the nth class, and may be integrated by Abel's Theorem.—Ed.]

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133. "When $x = 0$ find the value of $\dfrac{x - \sin x}{\tan x - x}$."

SOLUTION BY D. J. MC. ADAM, WASHINGTON, PA.

Taking differential coefficients of both numerator and denominator, we get $\dfrac{\cos^2 x}{1+\cos x}$, which, when $x=0$, is $\dfrac{1}{2}$.

[Solutions of 134, 135 and 136, will be published in No. 1, Vol. IV, as follows: 134, by Artemas Martin; 135, by E. B. Seitz; 136, by W. Hoover.]

PROBLEMS.

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137. BY L. REGAN.—A point D, is given in position between two lines which make a given angle at A. Find the position of a given line, BC, drawn through D, and intersecting the two lines in the points B and C.

138. BY PROF. J. SCHEFFER.—In a pentagon, $ABCDE$, the triangles $ABE = a$, $ADE = b$, $CDE = c$, $BCD = d$, $ABC = e$ are known; to find the area F of the pentagon.